Version 1.0



**General Certificate of Education (A-level)**
**June 2012**

# Mathematics MPC1

# (Specification 6360)

# Pure Core 1

***Mark Scheme***

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| –*x* EE | deduct *x* marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

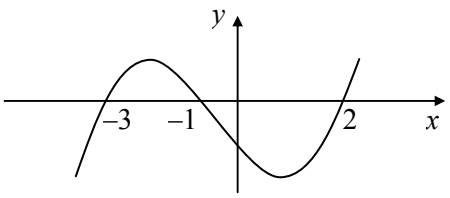
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MPC1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1** | $\frac{5\sqrt{3}-6}{2\sqrt{3}+3} \times \frac{2\sqrt{3}-3}{2\sqrt{3}-3}$ | M1 | | |
| | (Numerator $=$) $30-15\sqrt{3}-12\sqrt{3}+18$ | m1 | | correct $\left(=48-27\sqrt{3}\right)$ |
| | (Denominator $=12-9=$) $3$ | B1 | | must be seen as denominator |
| | $\left(\frac{48-27\sqrt{3}}{3}\right)=16-9\sqrt{3}$ | A1 | 4 | CSO; accept $16+-9\sqrt{3}$ |
| | **Total** | | **4** | |
| **2(a)(i)** | $y=\frac{4}{3}x-\frac{7}{3}$ | M1 | | $y=\pm\frac{4}{3}x+k$ or $\frac{\Delta y}{\Delta x}$ with 2 **correct** points |
| | $\Rightarrow$ grad $AB=\frac{4}{3}$ | A1 | 2 | condone slip in rearranging if gradient is correct; condone 1.33 or better |
| **(ii)** | $y=$ 'their grad' $x+c$ **and** attempt to use $x=3,\ y=-5$ | M1 | | ***or*** $y--5=$ 'their grad $AB$' $(x-3)$ ***or*** $4x-3y=k$ and attempt to find $k$ using $x=3$ and $y=-5$ |
| | $y+5=\frac{4}{3}(x-3)$ *or* $y=\frac{4}{3}x-\frac{27}{3}$ | A1 | | correct equation in any form but must simplify $--$ to $+$ |
| | $4x-3y=27$ | A1 | 3 | integer coefficients in required form eg $-8x+6y=-54$ |
| **(b)** | $4x-3y=7$ and $3x-2y=4$ | | | must use **correct pair** of equations and |
| | $\Rightarrow 8x-9x=14-12$ etc | M1 | | **attempt** to eliminate $x$ or $y$ (generous) |
| | $x=-2$ | A1 | | |
| | $y=-5$ | A1 | 3 | or $D\ (-2,-5)$ |
| **(c)** | $4(k-2)-3(2k-3)=7$ | | | sub $x=k-2,\ y=2k-3$ into $4x-3y=7$ |
| | $4k-8-6k+9=7$ | M1 | | and attempt to multiply out with all $k$ terms on one side (condone one slip) |
| | $\Rightarrow k=-3$ | A1 | 2 | |
| | **Total** | | **10** | |

**MPC1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)(i)** | $p(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$ | M1 | | $p(-1)$ attempted **not** long division |
| | $p(-1) = -1 + 2 + 5 - 6 = 0 \Rightarrow x + 1$ is a factor | A1 | 2 | CSO; correctly shown = 0 plus statement |
| **(ii)** | Quad factor in this form: $(x^2 + bx + c)$ | M1 | | long division as far as constant term ***or*** comparing coefficients, ***or*** $b = 1$ ***or*** $c = -6$ by inspection |
| | $x^2 + x - 6$ | A1 | | correct quadratic factor |
| | $[p(x) =]\ (x+1)(x+3)(x-2)$ | A1 | 3 | **must** see correct product |
| **(b)** | $p(0) = -6$ ; $p(1) = -8$ | M1 | | **both** $p(0)$ and $p(1)$ attempted **and** at least one value correct |
| | $\Rightarrow p(0) > p(1)$ | A1 | 2 | **AG** both values correct plus correct statement involving p(0) and p(1) |
| **(c)** |   | M1<br>A1<br>A1 | 3 | cubic with one max and one min<br>∿ with $-3, -1, 2$ marked<br>correct with minimum to right of $y$-axis AND going beyond –3 and 2 |
| | **Total** | | **10** | |

**MPC1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)(i)** | $3x^2+3x^2+xy+xy+3xy+3xy$ | M1 | | correct expression for surface area |
| | $6x^2+8xy=32$ | | | $2(3x^2+xy+3xy)=32$ etc |
| | $\Rightarrow 3x^2+4xy=16$ | A1 | 2 | **AG** be convinced |
| **(ii)** | $(V=)3x^2y$ OE | M1 | | correct volume in terms of $x$ and $y$ |
| | $=3x\left(\frac{16-3x^2}{4}\right)$ **or** $=3x^2\left(\frac{16-3x^2}{4x}\right)$ | | | OE |
| | $=12x-\frac{9x^3}{4}$ | A1 | 2 | CSO **AG**<br>be convinced that all working is correct |
| **(b)** | $\left(\frac{dV}{dx}=\right)12-\frac{27}{4}x^2$ | M1<br>A1 | 2 | one of these terms correct<br>all correct with $9\times 3$ evaluated<br>(no + $c$ etc) |
| **(c)(i)** | $x=\frac{4}{3}\Rightarrow\frac{dV}{dx}=12-\frac{27}{4}\times\left(\frac{4}{3}\right)^2$ | M1 | | attempt to sub $x=\frac{4}{3}$ into '*their*' $\frac{dV}{dx}$ |
| | $\frac{dV}{dx}=12-\frac{27}{4}\times\frac{16}{9}=12-12$ | | | or $12-\frac{432}{36}=12-12$ or $12-\frac{48}{4}=0$ etc |
| | $\frac{dV}{dx}=0\Rightarrow$ stationary value | A1 | 2 | CSO; shown = 0 plus statement |
| **(ii)** | $\frac{d^2V}{dx^2}=-\frac{27x}{2}$ OE | B1√ | | FT for '*their*' $\frac{dV}{dx}=a+bx^2$ |
| | when $x=\frac{4}{3}$, $\frac{d^2V}{dx^2}<0\Rightarrow$ maximum | E1√ | 2 | **or** sub of $x=\frac{4}{3}$ into *'their'* $\frac{d^2V}{dx^2}$<br>$\Rightarrow$ maximum |
| | (FT *"minimum"* if their $\frac{d^2V}{dx^2}>0$) | | | E0 if numerical error seen |
| | **Total** | | **10** | |

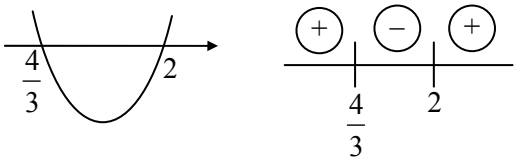
**MPC1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)(i)** | $\left(x-\frac{3}{2}\right)^2$ | M1 | | or $p = 1.5$ stated |
| | $\left(x-\frac{3}{2}\right)^2+\frac{11}{4}$ | A1 | 2 | $(x-1.5)^2+2.75$ |
| | ***Mark their final line as their answer*** | | | |
| **(ii)** | $x=\frac{3}{2}$ | B1√ | 1 | correct or FT their "$x = p$" |
| **(b)(i)** | $x^2-3x+5=x+5 \Rightarrow x^2=4x$ | M1 | | eliminating $x$ or $y$ and collecting like terms (condone **one** slip)<br>**or** $(y-5)^2-3(y-5)+5=y$<br>$\Rightarrow y^2-14y+45=0$ |
| | $(x\neq 0) \quad \Rightarrow x=4$ | A1 | | |
| | $y=9$ | A1 | 3 | |
| **(ii)** | $\frac{x^3}{3}-\frac{3x^2}{2}+5x\,(+c)$ | M1<br>A1<br>A1 | 3 | one of these terms correct<br>another term correct<br>all correct (need not have $+c$) |
| **(iii)** | $[\ ]_0^4=\frac{4^3}{3}-3\times\frac{4^2}{2}+5\times 4$ | M1 | | must have earned M1 in part(b)(ii)<br>F(their $x_B$) $\{-$F(0)$\}$ "correctly sub'd" |
| | $=17\frac{1}{3}$ | A1 | | $\left(\frac{64}{3}-24+20=\right)\ \frac{52}{3}$ or $\frac{104}{6}$ etc<br>condone 17.3 but not $16\frac{4}{3}$ etc |
| | Area trapezium $=\frac{1}{2}(x_B)(5+y_B)$ | B1√ | | FT their numerical values of $x_B, y_B$<br>Area = $\frac{1}{2}\times 4\times 14$ (= 28) |
| | Area of shaded region $=28-17\frac{1}{3}$ | | | |
| | $=10\frac{2}{3}$ | A1 | 4 | CSO; $\frac{32}{3}$, accept 10.7 or better |
| | **Total** | | **13** | |

**MPC1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)** | $(x-5)^2+(y-8)^2$ | B1 | | |
| | $=25$ | B1 | 2 | condone $5^2$ |
| **(b)(i)** | $(2-5)^2+(12-8)^2$ | | | or $AC^2=3^2+4^2$ |
| | $=9+16 \quad =25$ | | | hence $AC=5$ ; ( also radius = 5) |
| | $\Rightarrow A$ lies on circle | B1 | 1 | CSO<br>$(\Rightarrow$ radius $=AC) \Rightarrow A$ lies on circle |
| | (must have concluding statement and circle equation correct if using equation) | | | (must have concluding statement & RHS of circle equation correct or $r=5$ stated if Pythagoras is used) |
| **(ii)** | grad $AC=-\frac{4}{3}$ | B1 | | |
| | Gradient of tangent is $\frac{3}{4}$ | B1√ | | FT their –1/ grad $AC$ |
| | $y-12=$ *'their tangent grad'* $(x-2)$ | M1 | | **o**r $y=$ *'their tangent grad'* $x+c$<br>& attempt to find $c$ using $x=2$ , $y=12$ |
| | $y-12=\frac{3}{4}(x-2)$ ***or*** $y=\frac{3}{4}x+\frac{21}{2}$ etc | A1 | | correct equation in any form |
| | $3x-4y+42=0$ | A1 | 5 | CSO; must have integer coefficients with all terms on one side of equation<br>accept $0=8y-6x-84$ etc |
| **(c)(i)** | $(CM^2=) \quad (7-5)^2+(12-8)^2$ | M1 | | **or** $(CM^2=) \quad 20$ |
| | $(\Rightarrow CM=\sqrt{20}) \Rightarrow (CM=) \quad 2\sqrt{5}$ | A1 | 2 | |
| **(ii)** | $PM^2=PC^2-CM^2 \quad =25-20$ | M1 | | Pythagoras used correctly<br>eg $d^2+\left(2\sqrt{5}\right)^2=5^2$ |
| | $\Rightarrow PM=\sqrt{5}$ | A1 | | |
| | Area $\Delta PCQ=\sqrt{5}\times 2\sqrt{5}$<br>$=10$ | A1 | 3 | CSO |
| | **Total** | | **13** | |

**MPC1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)(i)** | $\left(\text{Increasing} \Rightarrow\right) \dfrac{dy}{dx} > 0$<br>$20x - 6x^2 - 16 > 0$ } either | M1 | | correct interpretation of $y$ increasing |
| | $\Rightarrow 6x^2 - 20x + 16 < 0$<br>*or* (2) $(10x - 3x^2 - 8) > 0$ | | | must see at least one of these steps before final answer for A1 |
| | $\Rightarrow 3x^2 - 10x + 8 < 0$ | A1 | 2 | CSO **AG** no errors in working |
| **(ii)** | $(3x-4)(x-2)$ | M1 | | correct factors or correct use of quadratic equation formula as far as $\dfrac{10 \pm \sqrt{4}}{6}$ |
| | CVs are $\dfrac{4}{3}$ and 2 | A1 | | condone $\dfrac{8}{6}$ and $\dfrac{12}{6}$ here but not in final line |
| |  | M1 | | sketch or sign diagram |
| | $\dfrac{4}{3} < x < 2$ | A1 | 4 | or $2 > x > \dfrac{4}{3}$<br>accept $x < 2$ **AND** $x > \dfrac{4}{3}$<br>but **not** $x < 2$ **OR** $x > \dfrac{4}{3}$<br>**nor** $x < 2$ , $x > \dfrac{4}{3}$ |
| | ***Mark their final line as their answer*** | | | |

**MPC1**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(b)(i)** | $x = 2$ ; $\left(\frac{dy}{dx} =\right) 40 - 24 - 16$ | M1 | | sub $x = 2$ into $\frac{dy}{dx}$ and simplify terms |
| | $\frac{dy}{dx} = 0 \Rightarrow$ tangent at $P$ is parallel to the $x$-axis | A1 | 2 | must be all correct working plus statement |
| **(ii)** | $x = 3$ ; $\frac{dy}{dx} = 20 \times 3 - 6 \times 3^2 - 16$ | M1 | | must attempt to sub $x = 3$ into $\frac{dy}{dx}$ |
| | $(= 60 - 54 - 16) \quad = -10$ | A1 | | |
| | Gradient of normal $= \frac{1}{10}$ | A1√ | | $\frac{-1}{"their\ -10"}$ |
| | Normal: $(y - -1) =$ 'their grad' $(x - 3)$ | m1 | | normal attempted with correct coordinates used and gradient obtained from their $\frac{dy}{dx}$ value |
| | $y + 1 = \frac{1}{10}(x - 3)$ | A1 | | any correct form, eg $10y = x - 13$ but must simplify $- -$ to $+$ |
| | (Equation of tangent at $P$ is ) $y = 3$ | B1 | | |
| | $x = 43$ | A1 | 7 | CSO; $\Rightarrow R(43, 3)$ |
| | **Total** | | **15** | |
| | **TOTAL** | | **75** | |